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Curvature Vacuum Correlations in *N*-Dimensional Einstein Gravity

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Under the flat Euclidean space–time background, the expressions for the leading terms of several two-point curvature vacuum correlation functions in N-dimensional Einstein gravity are calculated by using the perturbative expansion of the metric. It is shown that the contributions of the leading terms of such two-point curvature vacuum correlation functions are all vanishing.

KEY WORDS: metric perturbative expansion; vacuum correlations of curvature; N-dimensional einstein gravity.

1. INTRODUCTION

The metric determines the connection from which we can obtain the curvature of the space–time.

After the covariant quantization of the gravitational field, we can obtain the graviton propagator which is the two-point Green's function of the gravitational field. In terms of the propagating of gravity, we calculate the possible two-point transitions of the connection and the curvature, and formulate a valuable question for research (Guadagnini *et al.*, 1990; Modanese, 1992).

As for the two-point transition in curved space–time, it is a nontrivial physical question to construct an appropriate formulation of the curvature correlation function in vacuum. In recent years, the property of the holonomy which determines the parallel transport of vector in a space–time is conspicuous more and more. As we know, the Wilson loop that can be obtained from the trace of the holonomy is invariant under coordinate and gauge transformations. Thus, it often has been used to investigate gravity states (Bartolo *et al.*, 1995; Griego, 1996) and to construct "observable quantities" (Rovelli and Smolin, 1990, 1995) in nonperturbative quantum

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gravity theory. In perturbative quantum gravity, the parallel transport determined by the Wilson loop has been used to research the possible quantum behavior of excitation (Modanese, 1994) of the space–time curvature and to construct the two-point correlation functions of the space–time curvature.

By using the holonomy as the propagator of parallel transport of a vector along a geodesic, some forms of the invariant curvature correlation functions were defined in Modanese (1992). It is a valuable way to construct the correlation functions in curved space–time. In this paper, according to this way, under the Euclidean space–time background, we formulate several two-point curvature vacuum correlation functions and calculate their leading term contributions in the N-dimensional Einstein gravity one by one. It is shown that the leading term contributions of such correlation functions vanish. Thus, the gravity cannot propagate in vacuum. This is a reasonable result for the gravity.

In Section 2, the perturbative expansion of the curvature in N-dimensional Einstein gravity is calculated. In Section 3, the propagators of gravity are given. In Section 4, the curvature vacuum correlations functions are calculated. In Section 5, conclusion and discussion are given.

2. THE PERTURBATIVE EXPANSION OF THE CURVATURE IN *N*-DIMENSIONAL EINSTEIN GRAVITY

Let M be an N-dimensional space–time manifold, its geometry given by the metric $g_{\mu\nu}(x)$. The dynamics of the N-dimensional Einstein gravitational field is given by the action

$$S = \frac{1}{k^2} \int d^N x \sqrt{g(x)} R(x) \tag{1}$$

where $k^2 = 16\pi G$ (G is the Newtonian gravitational constant, and $g(x) = \det g_{\mu\nu}(x)$). The Christoffel connection of M is defined as

$$\Gamma^{\alpha}_{\mu\beta} = \frac{1}{2} g^{\alpha\lambda}(g_{\beta\lambda,\mu} + g_{\mu\lambda,\beta} - g_{\mu\beta,\lambda})$$

The Riemann curvature tensor is

$$R^{\alpha}_{\mu\beta\nu} = \Gamma^{\alpha}_{\mu,\beta\nu} - \Gamma^{\alpha}_{\mu\nu,\beta} + \Gamma^{\lambda}_{\mu\beta}\Gamma^{\alpha}_{\lambda\nu} - \Gamma^{\lambda}_{\mu\nu}\Gamma^{\alpha}_{\lambda\beta}$$

We decompose the metric tensor $g_{\mu\nu}(x)$ as

$$g_{\mu\nu}(x) = \delta_{\mu\nu} + \hbar k h_{\mu\nu}(x) \tag{2}$$

where η denotes the small parameter. The term $\delta_{\mu\nu}$ in (2) is interpreted as the classical background metric, while the term $h_{\mu\nu}(x)$ is regarded as a weak gravitation field which represents gravity propagating in the vacuum. The indices of $h_{\mu\nu}(x)$ are raised and lowered by the metric of Euclidean background $\delta^{\mu\nu}$ and $\delta_{\mu\nu}$, respectively. Also the inverse metric can be obtained as

$$g^{\mu\nu}(x) = \delta^{\mu\nu} - \hbar k h^{\mu\nu} + \hbar^2 k^2 h^{\mu\alpha} h_\alpha^\nu + O(h^3) \tag{3}$$

where $O(h^3)$ is a three-order small quantity in h.

Substituting (2) and (3) into (1), we obtain the expansion formula for the connection

$$\Gamma^\alpha_{\mu\beta} = \bar{\Gamma}^\alpha_{\mu\beta} - \bar{\bar{\Gamma}}^\alpha_{\mu\beta} + O(h^3) \tag{4}$$

where

$$\bar{\Gamma}^\alpha_{\mu\beta} = \frac{k}{2}(h_{\alpha\beta,\mu} + h_{\alpha\mu,\beta} - h_{\mu\beta,\alpha})$$

$$\bar{\bar{\Gamma}}^\alpha_{\mu\beta} = \frac{1}{2}\hbar^2 k^2 (h_{\alpha\lambda} h_{\lambda\beta,\mu} + h_{\alpha\lambda} h_{\lambda\mu,\beta} - h_{\alpha\lambda} h_{\mu\beta,\lambda})$$

By using (4), we can expand the curvature to a linear part and a square part in terms of h. We calculate the Riemann curvature tensor, the Ricci curvature tensor, the rotation matrix, and the curvature scalar.

(a) The Riemann curvature tensor

$$R^\alpha_{\mu\beta\nu} = \bar{R}^\alpha_{\mu\beta\nu} + \bar{\bar{R}}^\alpha_{\mu\beta\nu} + O(h^3) \tag{5}$$

where

$$\begin{aligned}\bar{R}^\alpha_{\mu\beta\nu} &= \bar{\Gamma}^\alpha_{\mu\beta,\nu} - \bar{\Gamma}^\alpha_{\mu\nu,\beta} \\ &= \frac{1}{2}\hbar k (h_{\alpha\mu,\beta,\nu} + h_{\alpha\beta,\mu,\nu} - h_{\beta\mu,\nu,\alpha} - h_{\alpha\mu,\nu,\beta} - h_{\alpha\nu,\mu,\beta} + h_{\mu\nu,\alpha,\beta})\end{aligned} \tag{6}$$

$$\begin{aligned}\bar{\bar{R}}^\alpha_{\mu\beta\nu} &= \bar{\bar{\Gamma}}^\alpha_{\mu\beta,\nu} - \bar{\bar{\Gamma}}^\alpha_{\mu\nu,\beta} + \bar{\Gamma}^\lambda_{\mu\beta}\bar{\Gamma}^\alpha_{\lambda\nu} - \bar{\Gamma}^\lambda_{\mu\nu}\bar{\Gamma}^\alpha_{\lambda\beta} \\ &= -\frac{1}{2}\hbar^2 k^2 [h_{\lambda\alpha}(h_{\mu\lambda,\beta} + h_{\beta\lambda,\mu} - h_{\beta\mu,\lambda})]_{,\nu} \\ &\quad + \frac{1}{2}\hbar^2 k^2 [h_{\alpha\lambda}(h_{\mu\lambda,\nu} + h_{\nu\lambda,\mu} - h_{\mu\nu,\lambda})]_{,\mu} \\ &\quad + \frac{1}{4}\hbar^2 k^2 (h_{\mu\lambda,\beta} + h_{\beta\lambda,\mu} - h_{\beta\mu,\lambda}) \cdot (h_{\alpha\lambda,\nu} + h_{\alpha\nu,\lambda} - h_{\lambda\nu,\alpha}) \\ &\quad - \frac{1}{4}\hbar^2 k^2 (h_{\mu\lambda,\nu} + h_{\nu\lambda,\mu} - h_{\mu\nu,\lambda}) \cdot (h_{\lambda\alpha,\beta} + h_{\alpha\beta,\lambda} - h_{\beta\lambda,\alpha})\end{aligned}$$

(b) The Ricci curvature tensor

$$R_{\mu\nu} = R^\alpha_{\mu\lambda\nu} = \bar{R}_{\mu\nu} + \bar{\bar{R}}_{\mu\nu} + O(h^3) \tag{7}$$

where

$$\bar{R}_{\mu\nu} = \bar{\bar{\Gamma}}^{\lambda}_{\mu\pi,\nu} - \bar{\bar{\Gamma}}^{\lambda}_{\mu\nu,\lambda}$$
$$= \frac{1}{2}\hbar k(h_{\mu\nu,\alpha,\alpha} + h_{\lambda\lambda,\mu,\nu} - h_{\nu\lambda,\mu,\lambda} - h_{\mu\lambda,\nu,\lambda}) \tag{8}$$

$$\bar{\bar{R}}_{\mu\nu} = \bar{\bar{\Gamma}}^{\rho}_{\mu\rho,\nu} - \bar{\bar{\Gamma}}^{\rho}_{\mu\nu,\rho} + \bar{\Gamma}^{\rho}_{\lambda\nu}\bar{\Gamma}^{\lambda}_{\mu\rho} - \bar{\Gamma}^{\lambda}_{\mu\nu}\bar{\Gamma}^{\rho}_{\lambda\rho}$$
$$= \frac{1}{2}\hbar^2 k^2(h_{\rho\lambda,\nu}h_{\mu\rho,\lambda} - h_{\rho\lambda,\nu}h_{\mu\lambda,\rho} - h_{\rho\lambda}h_{\lambda\rho,\mu,\nu}$$
$$+ h_{\rho\lambda}h_{\mu\rho,\lambda,\nu} + h_{\rho\lambda}h_{\lambda\nu,\mu,\rho} - h_{\rho\lambda}h_{\mu\nu,\lambda,\rho} + h_{\rho\lambda,\rho}h_{\mu\lambda,\nu}$$
$$+ h_{\rho\lambda,\rho}h_{\nu\lambda,\mu} - h_{\rho\lambda,\rho}h_{\mu\nu,\lambda} + h_{\rho\nu,\lambda}h_{\mu\lambda,\rho} - h_{\lambda\nu,\rho}h_{\mu\lambda,\rho})$$
$$= \frac{1}{4}\hbar^2 k^2(h_{\mu\nu,\lambda}h_{\rho\rho,\lambda} - h_{\rho\lambda,\nu}h_{\rho\lambda,\mu} - h_{\mu\lambda,\nu}h_{\rho\rho,\lambda} - h_{\nu\lambda,\mu}h_{\rho\rho,\lambda})$$

(c) The "rotation matrix"

$$R^{\mu}_{\nu} = R^{\mu}_{\nu\alpha\beta}\sigma^{\alpha\beta}$$
$$= \left[\bar{R}^{\alpha}_{\nu\alpha\beta} + \bar{\bar{R}}^{\mu}_{\nu\alpha\beta} + O(h^3)\right]\sigma^{\alpha\beta} \tag{9}$$

where $\sigma^{\alpha\beta}$ is an infinitesimal surface around x. Equation (9) can be noted as rotation matrix (Modanese, 1992).

(d) The curvature scalar

$$R = g^{\mu\nu}R_{\mu\nu} = \bar{R} + \bar{\bar{R}} + O(h^3) \tag{10}$$

where

$$\bar{R} = \hbar k(h_{\mu\mu,\nu\nu} - h_{\mu\nu,\mu\nu}) - \hbar^2 k^2 h_{\alpha\gamma}$$
$$\times (h_{\alpha\gamma,\beta\beta} - h_{\alpha\beta,\beta,\gamma} - h_{\beta\gamma,\alpha,\beta} + h_{\beta\beta,\alpha,\gamma}) \tag{11}$$

$$\bar{\bar{R}} = \frac{1}{4}\hbar^2 k^2(2h_{\mu\beta,\alpha}h_{\alpha\beta,\mu} - 3h_{\mu\beta,\alpha}h_{\mu\beta,\alpha} - 4h_{\mu\beta,\beta}h_{\alpha\alpha,\mu}$$
$$+ 4h_{\mu\beta,\beta}h_{\alpha\mu,\alpha} + h_{\alpha\alpha,\mu}h_{\beta\beta,\mu})$$

3. THE PROPAGATORS

3.1. The Propagators of Parallel Transport of Tensor

We define the vacuum correlation function by means of the vector parallel transport propagator (Modanese, 1992, 1993) along the geodesic in order to obtain the appropriate form of the invariant two-point curvature vacuum correlation function in the curved space–time M. Let a vector $\nu^{\mu'}(x')$ at the point x' parallel transport to a vector $\nu^{\mu}(x)$ at the point x. Then the parallel transport is determined by the holonomy

$$H^{\mu}_{\mu'}(x, x') = p \exp\left(\int_{x'}^{x} d\xi^{\alpha}\, \Gamma^{\mu}_{\alpha\mu'}(\xi)\right) \tag{12}$$

where $H^{\mu}_{\mu'}(x, x')$ is the propagator of parallel transport of the vector, and P means that the integral is computed along the geodesic. The propagator of parallel transport of tensor $T^{\mu\nu\cdots}(x)$ on M is given by

$$H^{\mu\nu\cdots}_{\mu'\nu'\cdots}(x, x') = H^{\mu}_{\mu'}(x, x') H^{\nu}_{\nu'}(x, x') \cdots \tag{13}$$

The indices of $H^{\mu}_{\mu'}(x, x')$ are raised and lowered by $g^{\mu'\nu'}(x')$ and $g_{\mu\nu}(x)$ respectively:

$$H^{\mu}_{\mu'}(x, x') g^{\mu'\nu'}(x') = H^{\mu\nu'}(x, x') \tag{14}$$

$$H^{\mu}_{\mu'}(x, x') g_{\mu\nu}(x) = H_{\nu\mu'}(x, x') \tag{15}$$

In order to require the propagators of parallel transport which are used in different invariant curvature vacuum correlations, we expand (12) and have

$$\begin{aligned} H^{\mu}_{\mu'}(x, x') = \delta^{\mu}_{\mu'} &+ \int_{\chi'}^{\chi} d\xi^{\alpha}\, \Gamma^{\mu}_{\alpha\mu'}(\xi) \\ &+ \frac{1}{2}\int_{x'}^{x} d\xi^{\alpha} \int_{x'}^{x} d\xi^{\beta}\, \Gamma^{\mu}_{\alpha\mu'}(\xi)\Gamma^{\mu}_{\beta\mu'}(\xi) + O(\Gamma^3) \end{aligned} \tag{16}$$

Using (13)–(16), (2), and (3), we can obtain the propagators of parallel transport which are used in different curvature correlation functions along the geodesic as follows:

(a) The propagator of parallel transport of the Riemann curvature tensor

$$H^{\alpha\beta\gamma\alpha'\beta'\gamma'}_{\mu\mu'} = H_{\mu\mu'} H^{\alpha\alpha'} H^{\beta\beta'} H^{\gamma\gamma'} = \overset{\circ}{H}{}^{\alpha\beta\gamma\alpha'\beta'\gamma'}_{\mu\mu'} = \overline{H}{}^{\alpha\beta\lambda\alpha'\beta'\gamma'}_{\mu\mu'} + O(h^2) \tag{17}$$

where

$$\overset{\circ}{H}{}^{\alpha\beta\gamma\alpha'\beta'\gamma'}_{\mu\mu'} = \delta_{\mu\mu'}\delta^{\alpha\alpha'}\delta^{\beta\beta'}\delta^{\gamma\gamma'} \tag{18}$$

$$
\begin{aligned}
\bar{H}^{\alpha\beta\gamma\alpha'\beta'\gamma'}_{\mu\mu'} &= \hbar k[\delta^{\alpha\alpha'}\delta^{\beta\beta'}\delta^{\gamma\gamma'}h_{\mu\mu'}(x) - \delta_{\mu\mu'}\delta^{\alpha\alpha'}\delta^{\beta\beta'}h^{\gamma\gamma'}(x') \\
&\quad - \delta_{\mu\mu'}\delta^{\alpha\alpha'}\delta^{\gamma\gamma}h^{\beta\beta'}(x') - \delta_{\mu\mu'}\delta^{\beta\beta'}\delta^{\gamma\gamma'}h^{\alpha\alpha'}(x')] \\
&\quad + \delta_{\mu\mu'}\delta^{\alpha\alpha'}\delta^{\beta\beta'}\int_{\chi'}^{\chi} d\xi^{P}\,\bar{\Gamma}^{\gamma\gamma'}_{\rho}(\xi) + \delta_{\mu\mu'}\delta^{\alpha\alpha'}\delta^{\gamma\gamma'} \\
&\quad \times \int_{\chi'}^{\chi} d\xi^{\rho}\,\bar{\Gamma}^{\beta\beta'}_{\rho}(\xi) + \delta_{\mu\mu'}\delta^{\beta\beta'}\delta^{\gamma\gamma'}\int_{\chi'}^{\chi} d\xi^{\rho}\,\bar{\Gamma}^{\alpha\alpha'}_{\rho}(\xi) \\
&\quad + \delta^{\alpha\alpha'}\delta^{\beta\beta'}\delta^{\gamma\gamma'}\int_{\chi'}^{\chi} d\xi^{\rho}\,\bar{\Gamma}^{\mu\mu'}_{\rho}(\xi)
\end{aligned}
$$

(b) The propagator of parallel transport of the Ricci curvature tensor

$$
H^{\alpha\beta\alpha'\beta'} = H^{\alpha\alpha'}H^{\beta\beta'} = \overset{\circ}{H}{}^{\alpha\beta\alpha'\beta'} = \bar{H}^{\alpha\beta\alpha'\beta'} + O(h^2) \tag{19}
$$

where

$$
\overset{\circ}{H}{}^{\alpha\beta\alpha'\beta'} = \delta^{\alpha\alpha'}\delta^{\beta\beta'} \tag{20}
$$

$$
\begin{aligned}
\bar{H}^{\alpha\beta\alpha'\beta'} &= \delta^{\alpha\alpha'}\left[-\hbar k h^{\beta\beta'}(x') + \int_{x'}^{x} d\xi^{\rho}\,\bar{\Gamma}^{\beta\beta'}_{\rho}(\xi)\right] \\
&\quad + \delta^{\beta\beta'}\left[-\hbar k h^{\alpha\alpha'}(x') + \int_{x'}^{x} d\xi^{\rho}\,\bar{\Gamma}^{\alpha\alpha'}_{\rho}(\xi)\right]
\end{aligned}
$$

(c) The propagator of parallel transport of the rotation matrix

$$
H^{\alpha\alpha'}_{\mu\mu'} = H_{\mu\mu'}H^{\alpha\alpha'} = \overset{\circ}{H}{}^{\alpha\alpha'}_{\mu\mu'} + \bar{H}^{\alpha\alpha'}_{\mu\mu'} + O(h^2) \tag{21}
$$

where

$$
\overset{\circ}{H}{}^{\alpha\alpha'}_{\mu\mu'} = \delta_{\mu\mu'}\delta^{\alpha\alpha'} \tag{22}
$$

$$
\begin{aligned}
\bar{H}^{\alpha\alpha'}_{\mu\mu'} &= \hbar k\delta^{\alpha\alpha'}h_{\mu\mu'}(x) - \hbar k\delta_{\mu\mu'}h^{\alpha\alpha'}(x') + \delta_{\mu\mu'}\int_{x'}^{x} d\xi^{\rho}\,\bar{\Gamma}^{\alpha\alpha'}_{\rho}(\xi) \\
&\quad + \delta_{\alpha\alpha'}\int_{x'}^{x} d\xi^{\rho}\,\bar{\Gamma}^{\mu\mu'}_{\rho}(\xi)
\end{aligned}
$$

3.2. The Graviton Free Propagator

To get the propagator of gravity in the N-dimensional Einstein gravity, we may consider the functional

$$
Z = \int d[g]\,\exp\left[\frac{i}{\hbar}\{S(g)\}\right]
$$

as the original generating functional for the path integral quantization of the gravitational field. However, for simplicity, here we use the familiar Feynman–DeWitt propagator:

$$\langle h_{\mu\nu}(x)h_{\alpha\beta}(y)\rangle = -C_{\rm N}\{\delta_{\mu(\alpha}\delta_{\beta)\nu} - [2/(N-2)]\delta_{\mu\nu}\delta_{\alpha\beta}\}(x-y)^{2-N} \qquad (23)$$

where C_N ($N \geq 3$) is a positive coefficient, and

$$C_N = \frac{2^{N-2}\pi^{N/2}}{(2\pi)^N}\frac{\Gamma(N/2-1)}{\Gamma(1)}$$

In the above expression, the symbol Γ is the Γ function.

4. THE CALCULATION OF THE CURVATURE VACUUM CORRELATION FUNCTIONS

4.1. Definition of the Curvature Vacuum Correlation Functions

Now, we consider the tensor propagator which parallel transports a tensor at point x' to a tensor at point x along the geodesic of length D which connects x' to x. For the "different curvatures," we may formulate the invariant two-point vacuum correlation functions as follows:

(a) The two-point vacuum correlation function of the Riemann curvature tensor

$$G_{\rm Riemann}(D) = \left\langle R^{\mu}_{\nu\alpha\beta}(x)H^{\nu\alpha\beta\nu'\alpha'\beta'}_{\mu\mu'}(x,x')R^{\mu'}_{\nu'\alpha'\beta'}(x')\right\rangle_{\circ} \qquad (24)$$

(b) The two-point vacuum correlation function of the Ricci curvature tensor

$$G_{\rm Ricci}(D) = \langle R_{\alpha\beta}(x)H^{\alpha\beta\alpha'\beta'}(x,x')R^{\alpha'\beta'}(x')\rangle_{\circ} \qquad (25)$$

(c) The two-point vacuum correlation function of the rotation matrix

$$G_{\rm Loop}(D,\sigma,\sigma') = \left\langle R^{\mu}_{\nu}(x)H^{\nu\nu'}_{\mu\mu'}(x,x')R^{\mu'}_{\nu'}(x')\right\rangle_{\circ} \qquad (26)$$

(d) The two-point vacuum correlation function of the curvature scalar

$$G_R(D) = \langle R(x)R(x')\rangle_{\circ} \qquad (27)$$

4.2. Calculation of the Correlation Functions

(a) The calculation of $G_{\rm Riemann}(D)$
Introducing (5) and (17) into (24), we get

$$G_{\rm Riemann}(D) = G^1_{\rm Riemann}(D) + G^2_{\rm Riemann}(D) + O(h^4)$$

where

$$G^1_{\text{Riemann}}(D) = \left\langle \overline{R}^{\mu}_{\nu\alpha\beta}(x)\overset{\circ}{H}{}^{\nu\alpha\beta\nu'\alpha'\beta'}_{\mu\mu'}(x,x')\overline{R}^{\mu'}_{\nu'\alpha'\beta'}(x)\right\rangle_{\circ} \tag{28}$$

$$\begin{aligned} G^2_{\text{Riemann}}(D) &= \left\langle \overline{\overline{R}}{}^{\mu}_{\nu\alpha\beta}(x)\overset{\circ}{H}{}^{\nu\alpha\beta\nu'\alpha'\beta'}_{\mu\mu'}(x,x')\overline{R}^{\mu'}_{\nu'\alpha'\beta'}(x')\right\rangle_{\circ} \\ &\quad + \left\langle \overline{R}^{\mu}_{\nu\alpha\beta}(x)\overset{\circ}{H}{}^{\nu\alpha\beta\nu'\alpha'\beta'}_{\mu\mu'}(x,x')\overline{\overline{R}}{}^{\mu'}_{\nu'\alpha'\beta'}(x')\right\rangle_{\circ} \\ &\quad + \left\langle \overline{R}^{\mu}_{\nu\alpha\beta}(x)\overset{\circ}{H}{}^{\nu\alpha\beta\nu'\alpha'\beta'}_{\mu\mu'}(x,x')\overline{R}^{\mu'}_{\nu'\alpha'\beta'}(x')\right\rangle_{\circ} \end{aligned}$$

Now, we calculate the first term $G^1_{\text{Riemann}}(D)$ of the correlation function of the Riemann curvature tensor. Putting (6) and (18) into (28), we get

$$\begin{aligned} G'_{\text{Riemann}}(D) &= \frac{1}{4}\hbar^2k^2\langle(h_{\alpha\mu,\nu,\beta}(x) - h_{\alpha\nu,\mu,\beta}(x) \\ &\quad - h_{\beta\mu,\nu,\alpha}(x) + h_{\nu\beta,\mu,\alpha}(x))\cdot(h_{\alpha\mu,\nu,\beta}(x') \\ &\quad - h_{\alpha\nu,\mu,\beta}(x') - h_{\beta\mu,\nu,\alpha}(x') + h_{\nu\beta,\mu,\alpha}(x'))\rangle_{\circ} \\ &= 4\times\frac{1}{4}\hbar^2k^2[\langle h_{\alpha\mu,\nu,\beta}(x)h_{\alpha\mu,\nu,\beta}(x')\rangle_{\circ} \\ &\quad - \langle h_{\alpha\mu,\nu,\beta}(x)h_{\alpha\nu,\mu,\beta}(x')\rangle_{\circ} - \langle h_{\alpha\mu,\nu,\beta}(x)h_{\beta\mu,\nu,\alpha}(x')\rangle_{\circ} \\ &\quad + \langle h_{\alpha\mu,\nu,\beta}(x)h_{\nu\beta,\mu,\alpha}(x')\rangle_{\circ}] \end{aligned} \tag{29}$$

Substituting (23) into (29) and using

$$\begin{aligned} \Box\frac{C_N}{(x-y)^{N-2}} &= \Box\int d^N[(2\pi)^{-N}P^{-2}]\cdot e^{ip(x-x')} \\ &= -\delta^N(x-y) \end{aligned}$$

where $\Box = \partial^{\mu}\partial_{\mu} = \partial_{\mu}\partial_{\mu}$.
We obtain the result

$$G^1_{\text{Riemann}}(D) = \hbar^2k\frac{N^3-3N^2+2}{N-2}\,\Box\,\delta^N(x-x') \tag{30}$$

(b) The calculation of $G_{\text{Ricci}}(D)$
Putting (7) and (18) into (25), we get

$$G_{\text{Ricci}}(D) = G^1_{\text{Ricci}}(D) + G^2_{\text{Ricci}}(D) + O(h^4)$$

where

$$G^1_{\text{Ricci}}(D) = \langle\overline{R}_{\alpha\beta}(x)\overset{\circ}{H}{}^{\alpha\beta\alpha'\beta'}(x,x')\overline{R}_{\alpha'\beta'}(x')\rangle_{\circ} \tag{31}$$

$$G^2_{\text{Ricci}}(D) = \langle \bar{\bar{R}}_{\alpha\beta}(x)\overset{\circ}{H}{}^{\alpha\beta\alpha'\beta'}(x,x')\bar{R}_{\alpha'\beta'}(x')\rangle_\circ + \langle \bar{R}_{\alpha\beta}(x)\overset{\circ}{H}{}^{\alpha\beta\alpha'\beta'}(x,x')\bar{\bar{R}}_{\alpha'\beta'}(x')\rangle_\circ + \langle \bar{R}_{\alpha\beta}(x)\bar{H}^{\alpha\beta\alpha'\beta'}(x,x')\bar{R}_{\alpha'\beta'}(x')\rangle_\circ$$

In order to obtain $G^1_{\text{Ricci}}(D)$, we introduce (8) and (20) into (31), and then we have

$$\begin{aligned} G^1_{\text{Ricci}}(D) &= \frac{1}{4}\hbar^2 k^2 \langle (h_{\alpha\beta,\gamma,\gamma}(x) + h_{\gamma\gamma,\alpha,\beta}(x) - h_{\beta\gamma,\alpha,\gamma}(x) \\ &\quad - h_{\alpha\gamma,\beta,\gamma}(x)) \cdot (h_{\alpha\beta,\gamma,\gamma}(x') + h_{\gamma\gamma,\alpha,\beta}(x') \\ &\quad - h_{\beta\gamma,\alpha,\gamma}(x') - h_{\alpha\gamma,\beta,\gamma}(x'))\rangle_\circ \\ &= \frac{1}{4}\hbar^2 k^2 [\langle h_{\mu\nu,\alpha,\alpha}(x) h_{\mu\nu,\beta,\beta}(x')\rangle_\circ + \langle h_{\mu\mu,\alpha\alpha}(x) h_{\nu\nu,\beta\beta}(x')\rangle_\circ \\ &\quad - 2\langle h_{\mu\mu,\alpha,\alpha}(x) h_{\lambda\nu,\lambda,\nu}(x')\rangle_\circ - 2\langle h_{\nu\lambda,\alpha,\alpha}(x) h_{\mu\nu,\mu,\lambda}(x')\rangle_\circ \\ &\quad + 2\langle h_{\nu\lambda,\mu,\lambda}(x) h_{\mu\alpha,\nu,\alpha}(x')\rangle_\circ] \end{aligned} \tag{32}$$

Putting (23) into (32), we get

$$G^1_{\text{Ricci}}(D) = \hbar^2 k \frac{N^3 - 3N^2 - 2N + 4}{4(N-2)} \Box\, \delta^N(x - x') \tag{33}$$

(c) The calculation of $G_{\text{Loop}}(D, \sigma, \sigma')$
By virtue of (9) and (21), we may get from (26)

$$G_{\text{Loop}}(D,\sigma,\sigma') = G^1_{\text{Loop}}(D,\sigma,\sigma') + G^2_{\text{Loop}}(D,\sigma,\sigma') + O(h^4) \tag{34}$$

where

$$G^1_{\text{Loop}}(D,\sigma,\sigma') = \left\langle \bar{R}^{\alpha}_{\beta\mu\nu}(x)\overset{\circ}{H}{}^{\beta\beta'}_{\alpha\alpha'}(x,x')\bar{R}^{\alpha'}_{\beta'\mu'\nu'}(x')\right\rangle_\circ \sigma^{\mu\nu}\sigma^{\mu'\nu'} \tag{35}$$

$$\begin{aligned} G^2_{\text{Loop}}(D,\sigma,\sigma') &= \left\langle \bar{\bar{R}}^{\alpha}_{\beta\mu\nu}(x)\overset{\circ}{H}{}^{\beta\beta'}_{\alpha\alpha'}(x,x')\bar{R}^{\alpha'}_{\beta'\mu'\nu'}(x')\right\rangle_\circ \sigma^{\mu\nu}\sigma^{\mu'\nu'} \\ &\quad + \left\langle \bar{R}^{\alpha}_{\beta\mu\nu}(x)\overset{\circ}{H}{}^{\beta\beta'}_{\alpha\alpha'}(x,x')\bar{\bar{R}}^{\alpha'}_{\beta'\mu'\nu'}(x')\right\rangle_\circ \\ &\quad + \left\langle \bar{R}^{\alpha}_{\beta\mu\nu}(x)\bar{H}^{\beta\beta'}_{\alpha\alpha'}(x,x')\bar{R}^{\alpha'}_{\beta'\mu'\nu'}(x')\right\rangle_\circ \end{aligned}$$

Using (6) and (22), we have from (35)

$$\begin{aligned} G^1_{\text{Loop}}(D,\sigma,\sigma') &= \frac{1}{4}\hbar^2 k^2 \sigma^{\mu\nu}\sigma^{\mu'\nu'} \langle (h_{\alpha\mu,\nu,\beta}(x) + h_{\nu\beta,\mu,\alpha}(x) \\ &\quad - h_{\mu\beta,\nu,\alpha}(x) - h_{\alpha\nu,\mu,\beta}(x')) \cdot (h_{\alpha\mu,\nu',\beta'}(x') \end{aligned}$$

$$
\begin{aligned}
&+ h_{\nu'\beta,\mu',\alpha}(x') - h_{\mu'\beta,\nu',\alpha}(x') - h_{\alpha\nu',\mu',\beta}(x'))\rangle_\circ \\
&= \frac{1}{4}\hbar^2 k^2 \sigma^{\mu\nu}\sigma^{\mu'\nu'}[\langle h_{\alpha\mu,\nu,\beta}(x) h_{\alpha\mu'\nu',\beta}(x')\rangle_\circ \\
&+ \langle h_{\alpha\mu,\nu,\beta}(x) h_{\nu\beta,\mu',\alpha}(x')\rangle_\circ - \langle h_{\mu\alpha,\nu,\beta}(x) \\
&\times h_{\mu'\beta,\nu',\alpha}(x')\rangle_\circ + \langle h_{\alpha\mu,\nu,\beta}(x) h_{\alpha\nu',\mu',\beta}(x')\rangle_\circ \\
&+ \langle h_{\nu\beta,\mu,\alpha}(x) h_{\mu',\alpha,\nu',\beta}(x')\rangle_\circ + \langle h_{\nu\beta,\mu,\alpha}(x) \\
&\times h_{\nu'\beta,\mu',\alpha}(x')\rangle_\circ + \langle h_{\nu\beta,\mu,\alpha}(x) h_{\mu',\beta,\nu',\alpha}(x')\rangle_\circ \\
&- \langle h_{\nu\beta,\mu,\alpha}(x) h_{\alpha\nu',\mu',\beta}(x')\rangle_\circ - \langle h_{\mu\beta,\nu,\alpha}(x) \\
&\times h_{\alpha\mu',\nu',\beta}(x')\rangle_\circ - \langle h_{\mu\beta,\nu,\alpha}(x) h_{\nu'\beta,\mu',\alpha}(x')\rangle_\circ \\
&+ \langle h_{\mu\beta,\nu,\alpha}(x) h_{\mu'\beta,\nu'\alpha}(x')\rangle_\circ + \langle h_{\mu\beta,\nu,\alpha}(x) \\
&\times h_{\alpha\nu',\mu',\beta}(x')\rangle_\circ - \langle h_{\alpha\nu,\mu,\beta}(x) h_{\alpha\mu'\nu',\beta}(x')\rangle_\circ \\
&- \langle h_{\alpha\nu,\mu,\beta}(x) h_{\nu'\beta,\mu',\alpha}(x')\rangle_\circ + \langle h_{\alpha\nu,\mu,\beta}(x) \\
&\times h_{\mu'\beta'\nu',\alpha}(x')\rangle_\circ + \langle h_{\alpha\nu,\mu,\beta}(x) h_{\alpha\nu',\mu',\beta}(x')\rangle_\circ]
\end{aligned} \tag{36}
$$

Introducing (23) into (36), we obtain

$$
G^1_{\text{Loop}}(D, \sigma, \sigma') = 2\hbar^2 k^2 \frac{N^2 - 2N - 2}{N - 2} \sigma^{\mu\nu}\sigma^\lambda_\nu \partial_\mu \partial_\lambda \delta^N(x - x') \tag{37}
$$

In our abelian approximation, the calculation result of $G^1_{\text{Loop}}(D, \sigma, \sigma')$ is identical to the one of a Wilson loop computed along a dumbbell-like contour.

(d) The calculation of $G_{\text{R}}(D)$

Putting (15) into (35), we get

$$
G_{\text{R}}(D) = G^1_{\text{R}}(D) + G^2_{\text{R}}(D) + O(h^4)
$$

where

$$
\begin{aligned}
G^1_{\text{R}}(D) &= \langle \bar{R}(x) \bar{R}(x')\rangle_\circ \\
G^2_{\text{R}}(D) &= \langle \bar{R}(x) \bar{\bar{R}}(x')\rangle_\circ + \langle \bar{\bar{R}}(x) \bar{R}(x')\rangle_\circ
\end{aligned} \tag{38}
$$

Using (11), and from (38), we have

$$
\begin{aligned}
G^1_{\text{R}}(D) &= \hbar^2 k^2 \langle (h_{\mu\mu,\nu,\nu}(x) - h_{\mu\nu,\mu,\nu}(x)) \cdot (h_{\mu\mu,\nu,\nu}(x') - h_{\mu\nu,\mu,\nu}(x'))\rangle_\circ \\
&= \hbar^2 k^2 [\langle h_{\mu\mu,\nu,\nu}(x) h_{\alpha\alpha,\beta,\beta}(x')\rangle_\circ - \langle (h_{\mu\mu,\nu,\nu}(x) h_{\alpha\beta,\alpha,\beta}(x')\rangle_\circ \\
&\quad - \langle h_{\mu\nu,\mu,\nu}(x) - h_{\alpha\alpha,\beta,\beta}(x')\rangle_\circ + \langle h_{\mu\mu,\nu,\nu}(x) h_{\alpha\beta,\alpha,\beta}(x')\rangle_\circ]
\end{aligned} \tag{39}
$$

By virtue of (22) and (39), the following result is obtained:

$$G^1_{\rm R}(D) = -\hbar^2 k^2 \frac{2(N-1)}{N-2} \Box \delta^N(x - x') \tag{40}$$

5. DISCUSSION

Expressions (30), (33), (37), and (40) show that all the leading terms of the correlation functions $G_{\rm Riemann}$, $G_{\rm Ricci}$, $G_{\rm Loop}$, and $G_{\rm R}$ contain the derivative factors of δ-function, which vanish in the case we consider. Thus, we can show that the contribution of the first term, of order $\hbar^2 k^2$, vanishes for every vacuum correlation function in the N-dimensional Einstein gravity. We conclude that the contribution of the first term of order $\hbar^2 k^2$ vanishes for every vacuum correlation function in the N-dimensional Einstein gravity. As for the higher order, contributions come from the second terms and the loop corrections may be nonvanishing, but their quantities are very small. In Einstein gravity, because of the mass dimension of the gravitational coupling constant k, after quantization of the gravity we shall get a non-renormalizable quantum theory. So the value of the higher order terms and the corrections is not so important as the value of the first term to each correlation function. Thus, under the current situation of perturbative quantization of the gravity, the two-point curvature correlation functions shall vanish (neglecting the higher order contributions). This result is consistent with many authors' estimation (Modanese, 1992, 1994).

If we consider the action of Einstein gravity under the flat Minkowski space–time background as

$$S = \frac{1}{k^2} \int d^N x \, \sqrt{-g(x)} g^{\mu\nu} R_{\mu\nu}$$

and expand the metric density perturbatively with the Minkowski metric $\eta_{\mu\nu}$, we have

$$\tilde{g}^{\mu\nu} = \sqrt{-g} g^{\mu\nu} \eta^{\mu\nu} + \hbar k h^{\mu\nu}$$

For the original generating functional, we set

$$Z = \int d[\tilde{g}] \exp\left[\frac{i}{\hbar}\{S(\tilde{g})\}\right]$$

Then the calculation results of the first terms remain identical to the results of (30), (33), (37), and (40), respectively.

If one defines the correlation functions as those given in this paper, and considers the renormalizable $R + R^2 + R^{\cdot\cdot} R_{\cdot\cdot}$-gravity, one can obtain the fact that the leading terms of the two-point vacuum correlation functions do not vanish. In the $R + R^2 + R^{\cdot\cdot} R_{\cdot\cdot}$-gravity, the curvature may be used as the counter-terms in the lagrangian to cancel the divergences in the theory (Stelle, 1977), and the curvature can be propagated in the gravity; this is consistent with that the curvature correlation functions of the gravity are not zero.

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